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A New Extended Exponentiated Exponential Distribution with Application to Truncated Life Tests

Braimah Joseph Odunayo[1*], Ibrahim Sule[2], Salisu Shehu Umar[3], Bukoye Abdulwasiu[4]and Edike Nnamdi[5]

[1,5]*Department of Mathematics and Statistics, Ambrose Alli University,P.M.B. 14, Ekpoma, Edo State, Nigeria.*
[2]*Department of Statistics, Faculty of Physical Sciences, Ahmadu Bello University, Zaria, Nigeria.*
[3,4]*Department of Statistics, Auchi Polytechnic, Auchi, Edo State, Nigeria.*

**Corresponding Author's Email: ojbraimah2014@gmail.com*

Abstract

In this paper, the Extended Exponentiated Exponential distribution was developed from the New Extended Exponentiated-G family of distributions. Some mathematical properties of the newly derived distribution such as moment, moment generating function, quantile function, hazard function, survival function, odd function, distribution of order statistic and maximum likelihood estimation were all derived. An acceptance sampling plan based on truncated life tests was also developed for the distribution. The minimum size, operating characteristic function values and minimum ratio of the sampling plan required to confirm the specified mean life are also presented. Data from 20 electric carts lifetime (months) utilized by a manufacturing company for delivery and domestic transportation services was used to illustrate the results. A comparison study with different sampling plans and distributions by earlier authors is also carried out to demonstrate the advantage of the new plan over existing plans. The findings revealed that the proposed plan is better and can be conveniently useful by researchers and Quality Control Personnel in product acceptance sampling inspection.

Keywords: Consumer's risk; Exponentiated Exponential; Operating characteristic function; Producer's risk; Truncated life test.

1. Introduction

Several generalized distributions have recently been developed with applications in health sciences, environmental science, engineering, and finance, among other domains. These applications show that the number of data sets with classical distributions is usually more rare than common. As a result,

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statisticians have made tremendous progress toward generalizing classical distributions and successfully applying them in applied areas.

For many years, researchers have been creating families of distributions, which are typically used to develop compound distributions. These compound probability distributions are predicted to be more flexible than existing ones and to suit data better. Some notable families of distribution are Topp Leone Kumaraswamy-G by Ibrahim et al. (2020), The Kumaraswamy-G by Cordeiro and Decastro (2011), Topp Leone-G by Al-Shomrani et al. (2016), Odd Lindley-G family by Gomes-Silva et al. (2017), Gompertz-G family by Alizadeh et al. (2017), Odd Frechet G family by Haq and Elgarhy (2018). There is, however, the need for a single distribution which could better model the various datasets captured in the aforementioned distributions, hence we proposed a generalization of the exponential distribution based on the family of distribution proposed by Elgarhy et al. (2017).

The exponential (Ex) distribution has been generalized by many authors. For example, Gupta and Kundu (2001) proposed the exponentiated Ex (EEx) distribution, Nadarajah and Kotz (2006) proposed the beta Ex (BEx) distribution, Cordeiro et al (2010) developed the Kumaraswamy Ex (KEx) distribution as a special case of the KumaraswamyWeibull distribution, and Cordeiro et al. (2010) proposed the KumaraswamyWeibull distribution. The transmuted generalized Ex (TGEx) distribution was developed by Khan et al. (2013), the alpha power Ex (APEx) distribution was studied by Mahdavi and Kundu (2016), and the Kumaraswamy transmuted Ex (KTEx) distribution was studied by Afify et al. (2016).

2. Materials and Methods

Elgarhy, et al. (2017) proposed the New Extended Exponentiated–G (NEET-G) family, which encompasses a broader range of distributions. The cumulative distribution function (cdf) and probability density function (pdf) for the NEET-G family are, respectively, given by:

$$F_{EtE-G}(x;\theta,\alpha,\lambda,\gamma)=\left\{1-\left[1-[G(x;\gamma)]\right]^{\alpha\lambda}\right\}^{\theta} \tag{1}$$

and

$$f_{EtE-G}(x;\theta,\alpha,\lambda,\gamma)=\theta\alpha\lambda g(x;\gamma)\left[1-[G(x;\gamma)]\right]^{\alpha\lambda-1}\left\{1-\left[1-[G(x;\gamma)]\right]^{\alpha\lambda}\right\}^{\theta} \tag{2}$$

where x is the variable being modeled and $\theta,\alpha,\lambda,\geq 0$ are the shape parameters. The cdf and pdf of exponential distribution are given respectively as:

$$G(x;\beta)=1-e^{-\beta x} \tag{3}$$

and

$$g(x;\beta)=\beta e^{-\beta x} \tag{4}$$

where $\beta\geq 0$ is the scale parameter.

The goal of this study is to propose the Extended Exponentiated Exponential (EEtEx) distribution. Study some of its properties like moments, quantile, moment generating function, hazard function, survival function, odd function, distribution of order statistic, estimate the parameters by means of the maximum likelihood method, and design a truncated life test on the EEtEx distribution and applying the tabulated results using real data sets.

2.1 The Proposed Extended Exponentiated Exponential (EEtEx) distribution

In this section, a new Extended Exponentiated Exponential (EEtEx) distribution is derived. To obtain the new distribution, (3) is inserted into (1) to give the cumulative distribution function (cdf) of EEtEx as:

$$F_{EEtEx}(x;\theta,\alpha,\lambda,\beta) = \left\{1-\left[e^{-\beta x}\right]^{\alpha\lambda}\right\}^{\theta} \tag{5}$$

On differentiating (5), we obtained the pdf of the new EEtEx distribution as:

$$f_{EEtEx}(x;\theta,\alpha,\lambda,\beta) = \theta\alpha\lambda\beta e^{-\alpha\lambda\beta x}\left\{1-e^{-\alpha\lambda\beta x}\right\}^{\theta-1} x,\theta,\alpha,\lambda,\beta \geq 0 \tag{6}$$

where $\beta \geq 0$is the scale parameter and $\theta, \alpha, \lambda, \geq 0$ are the shape parameters.

We can represent the distribution as follows using binomial expansion on (5) as:

$$\left\{1-\left[e^{-\beta x}\right]^{\alpha\lambda}\right\}^{\theta-1} = \sum_{i=0}^{\infty}\frac{(-1)^i\Gamma(\theta)}{i!\Gamma(\theta-i)}\left[e^{-\alpha\lambda\beta x}\right]^{i} \tag{7}$$

$$f_{EtEEx}(x;\theta,\alpha,\lambda,\beta) = \theta\alpha\lambda\beta\sum_{i=0}^{\infty}\frac{(-1)^i\Gamma(\theta)}{i!\Gamma(\theta-i)}\left[e^{-\alpha\lambda\beta x}\right]^{i+1} \tag{8}$$

Using (8), various properties of the EEtEx distribution can be derived.

2.2 Mathematical Properties

Some of the mathematical properties of the EEtEx family are derived and presented in this section.

2.2.1 Moments

$$E(x^r) = \int_0^{\infty} x^r f(x)dx \tag{9}$$

$$E(x^r) = \theta\alpha\lambda\beta\sum_{i=0}^{\infty}\frac{(-1)^i\Gamma(\theta)}{i!\Gamma(\theta-i)}\int_0^{\infty} x^r e^{-\alpha\lambda\beta x(i+1)}dx \tag{10}$$

where

$$\int_0^{\infty} x^r e^{-\alpha\lambda\beta x(i+1)}dx = \left(\frac{1}{\alpha\beta\lambda(i+1)}\right)^{r+1}\Gamma(r+1) \tag{11}$$

Therefore

$$E(x^r) = \theta\alpha^{-r}\beta^{-r}\lambda^{-r}\sum_{i=0}^{\infty}\frac{(-1)^i\Gamma(\theta)}{i!(i+1)^{r+1}\Gamma(\theta-i)}\Gamma(r+1) \tag{12}$$

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2.2.2 Moment generating function

The moment generating function (mgf) of X is obtained using the equation

$$M_x(t) = E(e^{tx}) = \int_0^\infty e^{tx} f(x)dx \tag{13}$$

$$e^{tx} = \sum_{m=0}^\infty \frac{t^m x^m}{m!} \tag{14}$$

$$E(e^{tx}) = \theta\alpha^{-m}\beta^{-m}\lambda^{-m} \sum_{m=0}^\infty \sum_{i=0}^\infty \frac{(-1)^i \Gamma(\theta) t^m}{i!m!(i+1)^{m+1}\Gamma(\theta-i)} \Gamma(m+1) \tag{15}$$

2.2.3 Quantile Function

The EEtEx distribution is simply approximated by inverting (5) as follows: If u has a uniform distribution $U(0,1)$, then the nonlinear equation's solution is:

$$\left\{1 - \left[e^{-\beta x}\right]^{\alpha\lambda}\right\}^\theta = u$$

$$u^{\frac{1}{\theta}} = 1 - \left[e^{-\beta x}\right]^{\alpha\lambda}$$

$$1 - u^{\frac{1}{\theta}} = \left[e^{-\beta x}\right]^{\alpha\lambda}$$

$$-\beta\lambda\alpha x = log\left[1 - u^{\frac{1}{\theta}}\right]$$

$$x = Q(u) = \frac{-log\left[1-u^{\frac{1}{\theta}}\right]}{\beta\lambda\alpha} \tag{16}$$

Equation (16) becomes the quantile function of Extended Exponentiated Exponential (EEtEx) distribution.

$$Q(0.5) = \frac{-log\left[1-0.5^{\frac{1}{\theta}}\right]}{\beta\lambda\alpha} \tag{17}$$

Equation (17) is the median of Extended Exponentiated Exponential (EEtEx) distribution.

2.2.4 Hazard function

$$H(x;\alpha,\beta,\lambda,\theta) = \frac{f(x;\alpha,\beta,\lambda,\theta)}{S(x,\alpha,\beta,\lambda,\theta)} \tag{18}$$

$$H(x;\alpha,\beta,\lambda,\theta) = \frac{\theta\alpha\lambda\beta e^{-\alpha\lambda\beta x}\left\{1-e^{-\alpha\lambda\beta x}\right\}^{\theta-1}}{1-\left\{1-\left[e^{-\beta x}\right]^{\alpha\lambda}\right\}^\theta} \tag{19}$$

2.2.5 Survival function

The survival function can be defined as the probability of an item not failing before a certain time (t). It is represented as:

$$S(x;\alpha,\beta,\lambda,\theta) = 1 - F(x;\alpha,\beta,\lambda,\theta) \tag{20}$$

$$S(x;\alpha,\beta,\lambda,\theta) = 1 - \left\{1 - \left[e^{-\beta x}\right]^{\alpha\lambda}\right\}^\theta \tag{21}$$

2.2.6 Odds function

The odds function is derived using the relation:

$$O(x;\alpha,\beta,\lambda,\theta)=\frac{F(x;\alpha,\beta,\lambda,\theta)}{S(x;\alpha,\beta,\lambda,\theta)} \tag{21}$$

$$O(x;\alpha,\beta,\lambda,\theta)=\frac{\left\{1-\left[e^{-\beta x}\right]^{\alpha\lambda}\right\}^{\theta}}{1-\left\{1-\left[e^{-\beta x}\right]^{\alpha\lambda}\right\}^{\theta}} \tag{22}$$

2.3 Distribution of Order statistic

Let $X_1, X_2, \ldots, X_n$ be n independent random variable from an Extended Exponentiated Exponential (EEtEx) distribution. Also, let $X_{(1)} \le X_{(2)} \le \ldots \le X_{(n)}$ be the corresponding order statistic. If $F_{r:n}(x)$ and $f_{r:n}(x)$, $r = 1,2,3,\ldots n$ represent the cdf and pdf of the r^{th} order statistics $X_{r:n}$ respectively, the pdf of the r^{th} order statistics of $X_{r:n}$ is given as

$$f_{r:n}(x;\alpha,\beta,\lambda,\theta)=\frac{1}{B(r,n-r+1)}\sum_{i=0}^{n-r}\frac{(-1)^i\Gamma(\mathrm{n-r+1})}{i!\Gamma(\mathrm{n-r+1-i})}\left[F(x;\alpha,\beta,\lambda,\theta)\right]^{r+i-1}f(x;\alpha,\beta,\lambda,\theta) \tag{23}$$

Using the cdf and pdf of Extended Exponentiated Exponential (EEtEx) distribution in (5) and (6), we have

$$f_{r:n}(x;\alpha,\beta,\lambda,\theta)=\frac{1}{B(r,n-r+1)}\sum_{j=0}^{\infty}\sum_{i=0}^{n-r}\frac{(-1)^{i+j}\Gamma(\theta(\mathrm{r+i}))}{i!j!\Gamma(\theta(\mathrm{r+i})-\mathrm{j})}\left[e^{-\beta\alpha\lambda x}\right]^{j+1} \tag{24}$$

Equation (24) becomes the r^{th} order statistic of the Extended Exponentiated Exponential (EEtEx) distribution. The minimum and maximum order-statistics are obtained by setting $r = 1$ and $r = n$ respectively in (24).

2.4 Maximum Likelihood Estimation (MLE)

$$logL = nlog\theta + nlog\alpha + nlog\lambda + nlog\beta - \alpha\beta\lambda\sum_{i=1}^{n}x_i + (\theta-1)\sum_{i=1}^{n}log\left[1-e^{-\alpha\lambda\beta x_i}\right] \tag{25}$$

$$\frac{\partial logL}{\partial\theta}=\frac{n}{\theta}+\sum_{i=1}^{n}log\left[1-e^{-\alpha\lambda\beta x_i}\right] \tag{26}$$

$$\frac{\partial logL}{\partial\alpha}=\frac{n}{\alpha}-\beta\lambda\sum_{i=1}^{n}x_i+(\theta-1)\sum_{i=1}^{n}\left[\frac{\alpha e^{-\alpha x_i}}{1-e^{-\alpha x_i}}\right] \tag{27}$$

$$\frac{\partial logL}{\partial\lambda}=\frac{n}{\lambda}-\beta\alpha\sum_{i=1}^{n}x_i+(\theta-1)\sum_{i=1}^{n}\left[\frac{\lambda e^{-\lambda x_i}}{1-e^{-\lambda x_i}}\right] \tag{28}$$

$$\frac{\partial logL}{\partial\beta}=\frac{n}{\beta}-\lambda\alpha\sum_{i=1}^{n}x_i+(\theta-1)\sum_{i=1}^{n}\left[\frac{\beta e^{-\beta x_i}}{1-e^{-\beta x_i}}\right] \tag{29}$$

Note: Equations (26), (27), (28) and (29) does not have their simple form and therefore, are intractable. Thus, the choice of non-linear estimations of parameters using an iterative procedure. The Extended Exponentiated Exponential (EEtEx) distribution's cdf, pdf, reliability, and Hazard functions graphs are shown in Fig. 1 to 4.

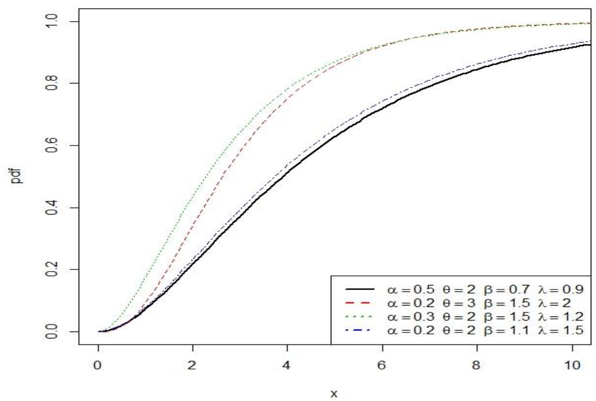

Figure1: Cdf Plot of Exponentiated Exponential (EEtEx) distribution

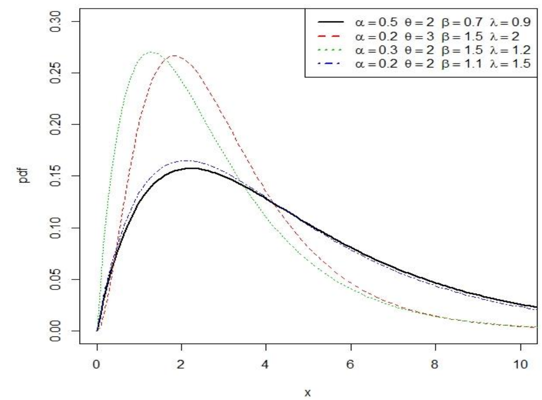

Figure 2: Pdf Plot of Exponentiated Exponential (EEtEx) distribution

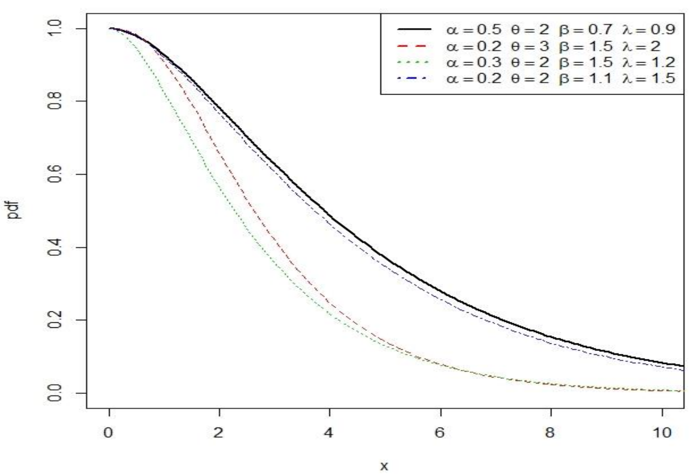

Figure 3: Survival Function Plot of Exponentiated Exponential (EEtEx) distribution

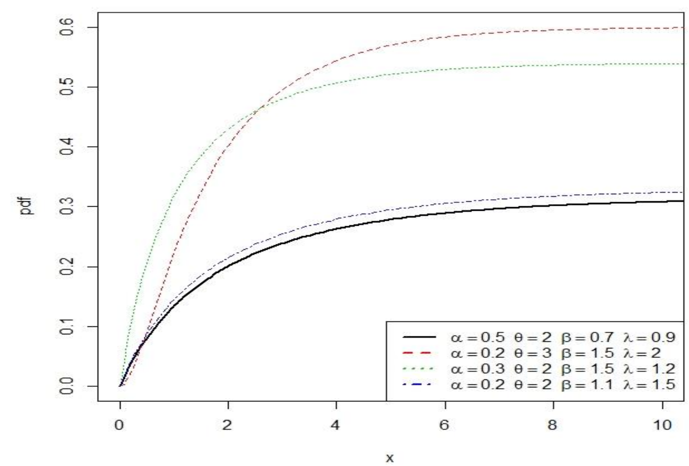

Figure 4: Hazard Function Plot of Exponentiated Exponential (EEtEx) distribution

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2.5 Acceptance Sampling Plan

In "statistical quality control" as a field, an acceptance sampling plan is a major tool for assuring quality. When testing of items is destructive and the cost of a 100 percent inspection is prohibitively very high, acceptance inspection is utilized (Al-Omari 2018). A random sample is randomly selected from the lot, and a decision is made whether to accept or reject the lot on the bases information provided by the sample. Because a product's lifetime of an item is expected to be very long and waiting until all of the goods fail might be time consuming, therefore it becomes a practice to end a life test at a predetermined time (t) in order to save time and money (Al-Omari 2016).

According to Mahdy et al. (2018), one major objective of this test is to create a confidence limit on the true mean life (μ) and a specified mean life (μ_0) with at least a probability of P^* (consumer's confidence level).

If the observed number of failures does not exceed a predefined acceptance number (c), the lot is accepted. If the number of failures reaches 'c', the test will be terminated and the lot discarded at time t_0. Using the truncated life test, the task at hand is to establish the minimum sample size (n) required to ensure a certain mean lifetime. A lot is considered acceptable if its true mean-life (μ) is more than the specified mean-life (μ_0), otherwise, it is rejected. For a given sample plan, the consumer's risk and the producer's risk are the probability of accepting bad lots and rejecting good lots, respectively (Sampath and Lalitha 2016).

In the rest of this study, acceptance sampling techniques based on a truncated life testing for the proposed Extended Exponentiated Exponential (EEtEx) distribution will be explored and applied to real-life data.

2.5.1 Design of the Truncated Sampling Plan

Let μ be the actual or true mean life of an item under consideration, and$t$, the length of timetheitem is subjected to a life test. The item's product life time is given by the ratio of the operational test time to the true mean life, $\frac{t}{\mu}$, which is considered the quality parameter. In this study, we are interested in modeling the item's product life time given by the ratio,$\frac{t}{\mu}$, hence we replace the variable x in the proposed distribution by the item's product life time $\frac{t}{\mu}$.

Assume that an item's product life time follows an Extended Exponentiated Exponential (EEtEx) distribution, the life distribution's cdf and pdf are as follows:

$$F_{EEtE\frac{t}{\mu}}\left(\frac{t}{\mu};\theta,\alpha,\lambda,\beta\right)=\left\{1-\left[e^{-\beta\frac{t}{\mu}}\right]^{\alpha\lambda}\right\}^{\theta} \tag{30}$$

and

$$f_{EEtE\frac{t}{\mu}}\left(\frac{t}{\mu};\theta,\alpha,\lambda,\beta\right)=\theta\alpha\lambda\beta e^{-\alpha\lambda\beta\frac{t}{\mu}}\left\{1-e^{-\alpha\lambda\beta\frac{t}{\mu}}\right\}^{\theta-1} \quad (31)$$

where $\frac{t}{\mu}\geq 0$ is the quality or scale parameter and $\theta,\alpha,\lambda,\beta\geq 0$ are the shape parameters.

The life test terminates at a predetermined time (t), and the number of failures is recorded between [0, t]. The lot is accepted if the number of failures at the end of time (t) is less than or equal to the acceptance number (c). The lot size is always assumed to be infinitely large in order for the binomial distribution to fit well. Assume that the consumer's risk is set to be at most 1- P*(where P* is the consumer's confidence level) (Malathi and Muthulakshmi 2017), or the probability that the true mean life is greater than 0 but not greater than *1-P**. The goal is to determine the smallest sample size (n) required to satisfy the inequality:

$$\sum_{i=0}^{c}\binom{n}{i}P^i(1-P)^{n-i}\leq 1-P^* \quad (32)$$

where c is the acceptance number of defective items for specified values of $P^*\in(0,1)$; $p=F(t;\mu_0)$, which is the probability that an item observed within the testing time (t) will fail depends only on the ratio $\frac{t}{\mu_0}$, where:

$$P=\left\{1-\left[e^{-\beta\frac{t}{\mu}}\right]^{\alpha\lambda}\right\}^{\theta}$$

and

$$\mu_0=E(x^1)=\theta\alpha^{-1}\beta^{-1}\lambda^{-1}\sum_{i=0}^{\infty}\frac{(-1)^i\Gamma(\theta)}{i!\,(i+1)^{1+1}\Gamma(\theta-i)}\Gamma(1+1)$$

$$\mu_0=\theta\alpha^{-1}\beta^{-1}\lambda^{-1}\sum_{i=0}^{\infty}\frac{(-1)^i\Gamma(\theta)}{i!(i+1)^2\Gamma(\theta-i)}\Gamma(2) \quad (33)$$

If the observed failures during testing time (t) is at most the acceptance number (c), we can conclude from (33) that $F(t;\mu)\leq F(t;\mu_0)$ with the probability P, which implies $\mu_0\leq\mu$ (Rady, Hassanein and Elhaddad 2016). The hypothesis (H_0:$\mu\geq\mu_0$) is accepted or rejected when the lot is accepted or rejected.

2.5.2 Minimum Sample Sizes

According to Braimah and Osanaiye (2017), the approximate of the improved sample size (n) is given as:

$$n=\left[\frac{\chi^2{}_{v,\beta}}{\rho F(t;\mu)}\right]+1 \quad (34)$$

We then calculate the smallest sample size values that satisfy inequality (32) for the ratio $\frac{t}{\mu_0}=$ 0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712 and P*= 0.75, 0.90, 0.95, 0.99 as presented in Table 1. The mean-ratio values $\left(\frac{t}{\mu_0}\right)$ and the confidence level (P*) are consistent with corresponding values of Wenhao and Shangli (2014), Amer, Nursel and Ayed (2020) and Amjad and Muhammad (2021), for the purpose of comparison.

Table 1: Minimum sample values to be selected for test from the lot for a specified time (t) to ascertain a probability P* and acceptance number (c) such that $\mu \geq \mu_0$

P*	C	$\frac{t}{\mu_0}$							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	4	3	3	2	2	2	2	2
0.75	1	6	5	4	4	3	3	3	3
0.75	2	9	7	6	5	5	4	4	4
0.75	3	11	9	8	7	6	5	5	5
0.75	4	14	11	9	8	7	6	6	6
0.75	5	16	13	11	9	8	7	7	7
0.75	6	19	14	12	11	9	8	8	8
0.75	7	21	16	14	12	10	10	9	9
0.75	8	23	18	15	14	11	11	10	10
0.75	9	26	20	17	15	13	12	11	11
0.75	10	28	22	18	16	14	13	12	12
0.90	0	5	4	3	3	2	2	2	2
0.90	1	8	6	5	5	4	3	3	3
0.90	2	11	8	7	6	5	5	4	4
0.90	3	14	11	9	8	6	6	5	5
0.90	4	17	13	10	9	8	7	6	6
0.90	5	19	15	12	11	9	8	7	7
0.90	6	22	16	14	12	10	9	8	8
0.90	7	24	18	15	14	11	10	9	9
0.90	8	27	20	17	15	12	11	11	10
0.90	9	29	22	19	16	14	12	12	11
0.90	10	32	24	20	18	15	13	13	12
0.95	0	6	5	4	3	3	2	2	2
0.95	1	10	7	6	5	4	4	3	3
0.95	2	13	10	8	7	5	5	4	4
0.95	3	16	12	10	8	7	6	6	5
0.95	4	18	14	11	10	8	7	7	6
0.95	5	21	16	13	11	9	8	8	7
0.95	6	24	18	15	13	10	9	9	8
0.95	7	27	20	16	14	12	10	10	9
0.95	8	29	22	18	16	13	12	11	10
0.95	9	32	24	20	17	14	13	12	12
0.95	10	34	26	21	19	15	14	13	13
0.99	0	9	7	5	5	3	3	3	2
0.99	1	13	9	8	6	5	4	4	4
0.99	2	16	12	10	8	6	5	5	5
0.99	3	19	14	12	10	8	7	6	6
0.99	4	22	16	13	12	9	8	7	7
0.99	5	25	19	15	13	10	9	8	8
0.99	6	28	21	17	15	12	10	9	9
0.99	7	31	23	19	16	13	11	10	10
0.99	8	34	25	21	18	14	12	11	11
0.99	9	37	27	22	19	15	13	13	12
0.99	10	39	29	24	21	17	15	14	13

2.5.3 Operating Characteristic

The operating characteristic function (probabilities of acceptance) of a given sampling plan (n, c, $\frac{t_0}{\mu_0}$) is the probability of accepting a lot when the number of failure after testing (Gui and Aslam, 2017). It is always considered as the basis for choosing the minimum sample (n) size and the acceptance number (c). The operating characteristic (assuming binomial function) of this acceptance sampling plan is defined as:

$$P_r(P) = P \text{ (Accepting a lot, given that } \mu < \mu_0) = \sum_{i=0}^{c}\binom{n}{i}P^i(1-P)^{n-i} \quad (35)$$

where $p = F\ ((t_0;\ \mu)$. The probabilities of acceptance or operating characteristic values for the Extended Exponentiated Exponential (EEtEx) distribution are presented in Table 2.

Table 2: Operating characteristic (Probability of Acceptance) values for sampling plan n, Acceptance Number (c) = 2, $\frac{t}{\mu_0}$ with specified probability P*

P*	n	t/μ∘	μ/μ∘					
			2	4	6	8	10	12
0.75	**9**	**0.628**	0.6111	0.8878	0.9548	0.9776	0.9874	0.9922
0.75	**7**	**0.942**	0.5914	0.8769	0.9492	0.9745	0.9855	0.9909
0.75	**6**	**1.257**	0.5709	0.8661	0.9436	0.9713	0.9835	0.9897
0.75	**5**	**1.571**	0.6341	0.8923	0.9553	0.9774	0.9870	0.9919
0.75	**5**	**2.356**	0.3887	0.7690	0.8924	0.9417	0.9650	0.9774
0.75	**4**	**3.141**	0.5258	0.8474	0.9325	0.9643	0.9789	0.9864
0.75	**4**	**3.927**	0.3793	0.7710	0.8929	0.9413	0.9643	0.9766
0.75	**4**	**4.712**	0.2615	0.6895	0.8474	0.9136	0.9463	0.9643
0.90	**11**	**0.628**	0.4472	0.8094	0.9175	0.9576	0.9755	0.9846
0.90	**8**	**0.942**	0.4703	0.8184	0.9211	0.9592	0.9764	0.9851
0.90	**7**	**1.257**	0.4173	0.7859	0.9035	0.9491	0.9700	0.9809
0.90	**6**	**1.571**	0.4356	0.7959	0.9079	0.9513	0.9713	0.9817
0.90	**5**	**2.356**	0.3888	0.7690	0.8924	0.9417	0.9650	0.9774
0.90	**5**	**3.141**	0.2126	0.6343	0.8126	0.8924	0.9328	0.9553
0.90	**4**	**3.927**	0.3793	0.7710	0.8929	0.9413	0.9643	0.9766
0.90	**4**	**4.712**	0.2615	0.6895	0.8474	0.9136	0.9463	0.9643
0.95	**13**	**0.628**	0.3127	0.7216	0.8710	0.9312	0.9593	0.9740
0.95	**10**	**0.942**	0.2770	0.6879	0.8502	0.9184	0.9510	0.9684
0.95	**8**	**1.257**	0.2734	0.6988	0.8553	0.9209	0.9524	0.9693
0.95	**7**	**1.571**	0.2815	0.6880	0.8479	0.9160	0.9491	0.9669
0.95	**5**	**2.356**	0.3887	0.7690	0.8924	0.9417	0.9650	0.9774
0.95	**5**	**3.141**	0.2126	0.6343	0.8126	0.8924	0.9328	0.9553
0.95	**4**	**3.927**	0.3793	0.7710	0.8929	0.9413	0.9643	0.9766
0.95	**4**	**4.712**	0.2615	0.6895	0.8474	0.9136	0.9463	0.9643
0.99	**16**	**0.628**	0.1714	0.5862	0.7889	0.8813	0.9274	0.9526
0.99	**12**	**0.942**	0.1522	0.5556	0.7662	0.8659	0.9168	0.9452
0.99	**10**	**1.257**	0.1331	0.5257	0.7433	0.8499	0.9058	0.9373
0.99	**8**	**1.571**	0.1739	0.5796	0.7793	0.8729	0.9209	0.9477
0.99	**6**	**2.356**	0.1946	0.6077	0.7959	0.8825	0.9267	0.9513
0.99	**5**	**3.141**	0.2126	0.6343	0.8126	0.8924	0.9328	0.9553
0.99	**5**	**3.927**	0.1056	0.5044	0.7243	0.8335	0.8923	0.9265
0.99	**5**	**4.712**	0.0486	0.3887	0.6342	0.7690	0.8459	0.8923

2.5.4 Minimum Ratio Function

The producer's risk (that is, PR) is the probability of rejecting a lot when it is good, that is, $\mu > \mu_0$. It is defined mathematically as (Rao, Ghitany and Kantam 2008):

$$PR = P \text{ (Rejecting a lot)} = \sum_{i=c+1}^{n}\binom{n}{i}P^i(1-P)^{n-i} \quad (36)$$

For this truncated sampling plan and any specified value for a producer's risk (R) that will affirm the producer's risk to be at most R. Since $P = F\left(\frac{t}{\mu_0}\frac{\mu_0}{\mu}\right)$ is a function of $\frac{\mu_0}{\mu}$, then $\frac{\mu_0}{\mu}$ becomes the smallest positive value for which P satisfies the given inequality below:

$$\sum_{i=c+1}^{n}\binom{n}{i}P^i(1-P)^{n-i} \leq R \quad (37)$$

where $P = \left\{1 - \left[e^{-\beta\frac{t}{\mu}}\right]^{\alpha\lambda}\right\}^{\theta}$.

For any specified value of producer's risk, for example γ, under any inspection plan, it may be of an experimenter interest to know the smallest the ratio value $\left(\frac{\mu}{\mu_0}\right)$ that will satisfies the producer's risk at most γ. This is the smallest positive number for $P = F\left(\frac{t}{\mu_0}\frac{\mu_0}{\mu}\right)$ to satisfies the inequality in equation (37). The Minimum Ratio Function values for Extended Exponentiated Exponential (EEtEx) distribution are presented in Table 3.

Table 3: Minimum ratio values for lots acceptability with producer's risk 0.05

P^*	c	t/μ_0							
		0.628	**0.942**	**1.257**	**1.571**	**2.356**	**3.141**	**3.972**	**4.713**
0.75	**0**	7.4200	9.6990	10.6030	13.2520	13.8780	18.5020	23.3980	27.7560
0.75	**1**	3.8550	4.3530	5.0060	6.2570	7.5910	10.1210	8.8630	10.5150
0.75	**2**	2.8500	3.4570	3.7200	3.9890	4.8220	6.4290	8.1300	6.7100
0.75	**3**	2.5170	2.8280	3.1460	3.4850	4.4760	4.8130	6.0850	7.2190
0.75	**4**	2.2360	2.4720	2.8170	3.1860	3.6230	3.9030	4.9350	5.8540
0.75	**5**	2.1220	2.3760	2.6020	2.7000	3.0730	4.0960	4.1950	4.9760
0.75	**6**	1.9830	2.1930	2.4490	2.6070	3.1310	3.5810	3.6760	4.3610
0.75	**7**	1.8780	2.0570	2.1830	2.3250	2.7940	3.1990	3.2920	3.9060
0.75	**8**	1.8410	2.0350	2.1140	2.2930	2.5320	2.9030	3.6710	3.5530
0.75	**9**	1.7700	1.9420	2.0560	2.2650	2.6180	2.6670	3.3720	3.2720
0.75	**10**	1.7140	1.8660	2.0080	2.0940	2.4240	2.8690	3.1280	3.0420
0.90	**0**	8.9670	11.1300	12.9420	16.1750	19.8740	26.4950	23.3970	27.7570
0.90	**1**	4.6000	5.3510	5.8090	7.2600	9.3840	10.1210	12.7980	15.1830
0.90	**2**	3.4440	4.0200	4.6130	5.2350	5.9820	6.4290	8.1300	9.6440
0.90	**3**	2.9280	3.2380	3.7740	3.9320	5.2260	5.9670	6.0850	7.2190
0.90	**4**	2.6340	2.9430	3.2990	3.5210	4.2310	4.8310	6.1080	5.8550
0.90	**5**	2.3810	2.7480	2.9910	3.2510	3.5850	4.0960	5.1800	4.9760
0.90	**6**	2.2550	2.5100	2.7750	3.0600	3.5360	4.1750	4.5290	5.3720
0.90	**7**	2.1590	2.3330	2.6140	2.7300	3.1530	3.7240	4.0450	4.7980
0.90	**8**	2.0460	2.2720	2.4890	2.6420	3.1570	3.3760	3.6710	4.3550
0.90	**9**	1.9900	2.1560	2.3880	2.5700	2.8930	3.0970	3.9160	4.0000

0.90	**10**	1.9120	2.1210	2.3060	2.3770	2.6770	3.2310	3.6280	3.7100
0.95	**0**	10.2290	12.3600	14.8510	16.1750	19.8740	26.4960	33.5060	39.7480
0.95	**1**	5.0270	5.7820	6.5110	7.2600	9.3840	12.5110	12.7980	15.1820
0.95	**2**	3.7040	4.2740	5.0020	5.2350	6.9720	7.9740	10.0840	9.6440
0.95	**3**	3.1130	3.6050	4.0560	4.3400	5.2260	5.9670	7.5460	7.2190
0.95	**4**	2.8460	3.2220	3.5180	3.8310	4.7780	5.6400	6.1090	7247.0000
0.95	**5**	2.5580	2.8620	3.1710	3.5020	4.0500	4.7800	5.1800	6.1450
0.95	**6**	2.4030	2.7040	2.9270	3.2680	3.9100	4.1740	5.2790	5.3720
0.95	**7**	2.2870	2.5850	2.7450	3.0950	3.4860	4.2030	4.7100	4.7980
0.95	**8**	2.1970	2.4210	2.6040	2.8040	3.4390	3.8070	4.2680	4.3550
0.95	**9**	2.1250	2.3540	2.4920	2.7130	3.1520	3.4900	3.9160	4.6460
0.95	**10**	2.0360	2.2410	2.4000	2.6380	2.9150	3.2310	3.6280	4.3040
0.99	**0**	12.2750	15.3440	17.9490	20.6130	24.2570	32.3400	33.5050	39.7480
0.99	**1**	5.9450	6.8990	8.2470	8.9250	10.8880	12.5110	15.8210	18.7690
0.99	**2**	4.3830	5.1660	5.7040	6.2520	7.8500	9.2950	10.0840	11.9630
0.99	**3**	3.6100	4.2460	4.8100	5.0690	6.5070	6.9660	8.8100	8.9520
0.99	**4**	3.2290	3.7230	4.1170	4.3980	6.5080	6.3700	7.1330	8.4610
0.99	**5**	2.9310	3.2840	3.6670	3.9630	4.8760	5.3980	6.0440	7.1700
0.99	**6**	2.7210	3.0590	3.3490	3.6570	4.2590	5.2130	5.9610	6.2630
0.99	**7**	2.5640	2.8910	3.2290	3.4300	4.0930	4.6480	5.3160	5.5870
0.99	**8**	2.4420	2.7610	3.0330	3.2550	3.7070	4.2090	4.8140	5.0640
0.99	**9**	2.3430	2.5980	2.8770	3.1140	3.6300	4.2020	4.4140	5.2360
0.99	**10**	2.2630	2.5180	2.7490	2.9990	3.3590	3.8870	4.5120	4.8470

3. Application to Real Data

The suggested Acceptance Sampling Plan (ASP) is illustrated using data from 20 electric carts lifetime (months) utilized by a manufacturing company for delivery and domestic transportation services in a big production plant. The values are: 1.5, 0.9, 2.3, 3.2, 6.2, 7.5, 8.3, 5.0, 3.9, 10.4, 12.6, 15.0, 16.3, 11.1, 22.6, 19.3, 24.8, 38.1, 31.5 and 53.0 (Al-Omari 2018). To start, we must determine whether the Extended Exponentiated Exponential (EEtEx) distribution fits the data. The empirical and theoretical density functions (pdf), theoretical distribution function (cdf), quantile (Q-Q), and probability (P-P) plots were all utilized to see if the data fit the theoretical distribution. The findings of the tests of goodness of fit were acceptable, indicating that the underlined distribution was well-fit.

Therefore, the estimated mean life of the 20 small electric carts is given as:

$$\mu_0 = \theta\alpha^{-1}\beta^{-1}\lambda^{-1}\sum_{i=0}^{\infty}\frac{(-1)^i\Gamma(\theta)}{i!(i+1)^2\Gamma(\theta-i)}\Gamma(2) = 14.68\text{months} \approx 15\text{months}.$$

Assuming the company want to put the small electric carts for a period of time (t = 14months). Therefore, $\frac{t}{\mu_0} = \frac{14}{15} = 0.9333$. Table 1 above shows that the minimum sample of items to be selected for operational test for a time (t) for t. Based on Table 1's estimated minimum sample values for $\frac{t}{\mu_0} = 0.9333$. Table 4 shows how we re-evaluated the minimal sample sizes for this experiment.

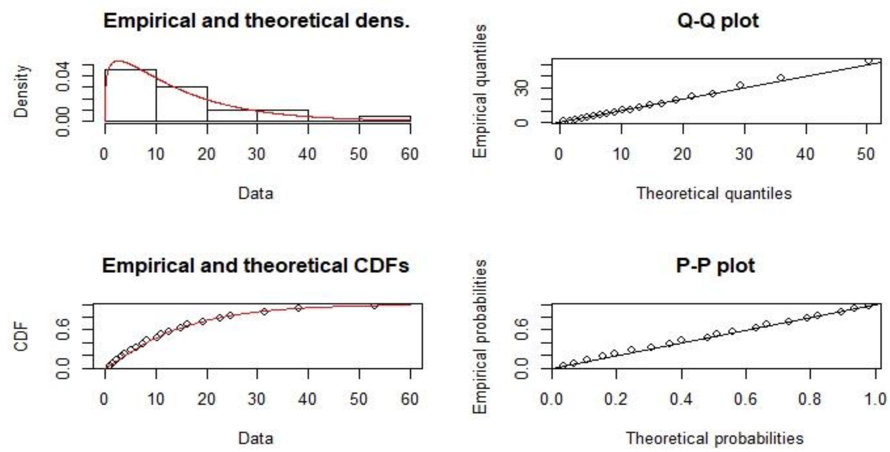

Figure 5: Fitted cdf, pdf, Q-Q and P-P Plot of the data set

Table 4: Electric Carts Minimum Sample Sizes

	C	0	1	2	3	4	5	6	7	8	9	10
	0.75	3	5	7	9	11	13	14	16	18	20	22
P^*	**0.90**	4	6	8	11	13	15	16	18	20	22	24
	0.95	4	7	10	12	14	16	18	20	22	24	26
	0.99	7	9	12	14	16	19	21	23	25	27	29

For example, from Table 4, a preferred ASP should be one with the highest consumer's confidence level i.e. corresponding to P* = 0.99 and $\frac{t}{\mu_0} = 0.9333$. If we decide to reject a sample having more than two failed items before time *t*, (i.e. acceptance number,c = 2), table 4 shows the minimum sample size for this Plan to be 12, hence the ideal acceptance sampling plan becomes ASP (12, 2, 0.933). On the bases of this information provided, the manufacturing firm will only need to purchase 12 Electric Carts (machines) in order to finish the manufacturing process in 14 months, even if 2 of the 12 machines have mechanical faults throughout the manufacturing process with probability of 0.99.

The probability of acceptance (operating characteristic) for the sampling plan is ASP (12, 2, 0.933) from Table 2 when the confidence level (P* =0.99) are given in Table 5.

Table 5: Probability of Acceptance for Proposed Plan on Electric Cart Data

$\frac{\mu}{\mu_0}$	2	4	6	8	10	12
OC	0.1522	0.5556	0.7662	0.8659	0.9168	0.9452
Producer's Risk	0.8478	0.4444	0.2338	0.1341	0.0832	0.0508

If the actual or true mean life of a product is twice the specified mean life$\left(\text{i. e.} \frac{\mu}{\mu_0} = 2\right)$, Table 5 shows that the producer's risk will be about 0.8478, 0.4444, 0.2338, 0.1341, 0.0832 and 0.0508 for

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$\frac{\mu}{\mu_0} = 2, 4, 6, 8, 10$ and 12 respectively. Therefore, the producer's risk is tends to zero as the true mean life becomes large.

Table 3 depicts the minimum ratio values of true mean lifetime to the specified mean lifetime for this sampling plan of a lot with a producer's risk R = 0.05.

For illustration, when P^* = 0:99 (that is, the consumer's risk will be 0.01), c = 2 and $\frac{t}{\mu_0} = 0.9333$, $\frac{\mu}{\mu_0} = 5.1660$, which implies that the true mean life of an Electric Cart will have to be $\mu = 5.1660 \times \mu_0 = 75.8369 \approx 76 months$, for it to be purchased and accepted with at least probability 0.95 and rejected with a probability that is less than or equal 0.05.

4. Comparative Analysis

The advantages of the proposed sampling plan for Extended Exponentiated Exponential (EEtEx) distribution is compared with other plans under different types of distributions assuming the true mean is five times the specified mean life and the acceptable number (c) of defectives equal two. The producer's risk is used as a comparison criterion. In comparison to other sample plans, one with a minimum value of the producer's risk is more efficient in reducing inspection costs. The proposed sampling plan is compared with other plans that were proposed by Amjad and Muhammad (2021) for power Lomax distribution, Amer et al. (2020) for Akash Distribution and Wenhao and Shangli (2014) for Gompertz Distribution.

Table 6 and Fig. 6 shows the comparison of results, which evident that the proposed sample plan has a lower Poducer's Risk and converges to zero than the other plans. These favourable results indicate that the new inspection sampling plan is better than the other three sampling plans included in these comparisons, using the electric cart data. It is therefore recommended for consideration by decision-makers and industrialists using other survival lifetime data.

Table 6: Comparative Sampling Plans ($n = 12,\ c = 2, \frac{t}{\mu_0} = 0.933$) when $\frac{\mu}{\mu_0} = 4$ and P^* =0.99

	$\frac{\mu}{\mu_0}$	2	4	6	8	10	12
Producer's Risk	Proposed Sampling Plan	0.8478	0.4444	0.2338	0.1341	0.0832	0.0508
	Amjad and Muhammad (2021)	0.8714	0.4536	0.2488	0.1398	0.0841	0.0519
	Amer et al. (2020)	0.8669	0.4743	0.2567	0.1500	0.0942	0.0743
	Wenhao and Shangli (2014)	0.8521	0.4302	0.2334	0.1502	0.0943	0.0740

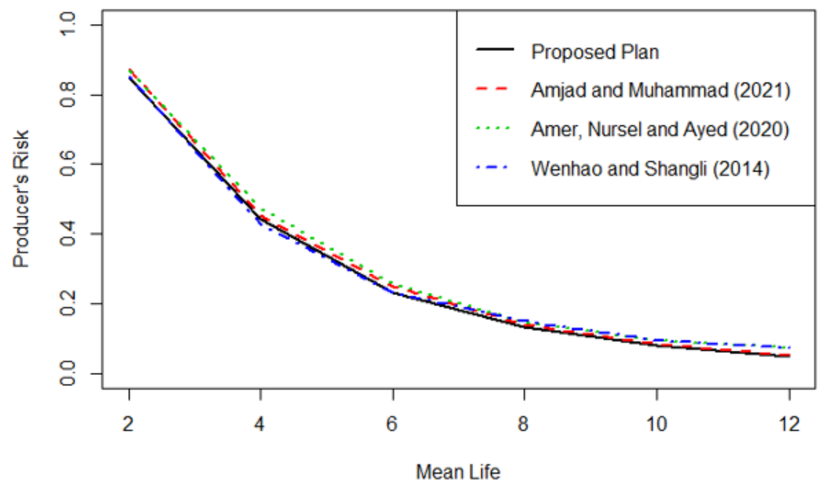

Figure 6: Producer's Risk Plot for Comparison of Sampling Plans

5. Conclusion

A new distribution was proposed in this study, with its application to acceptance sampling plans based on truncated life tests. Mathematical properties of the distribution were determined with the help of relevant plots. The relevant tables for minimum sample size required to ensure the mean life of the test units are presented. The probability of acceptance (operating characteristic) values are also presented, as well as the related producer risks. The proposed sampling plan was also applied to a real life data for illustration purpose. For the newly developed Extended Exponentiated Exponential (EEtEx) distribution and other distributions, the results of this study can be utilized to develop different types of acceptance sampling plans, like double, chain, and group acceptance sampling plans.

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